BULGARIAN ACADEMY OF SCIENCES
SPACE RESEARCH IN BULGARIA, 3
Sofia · 1980

A Solution of the Problem of Rotation of the Celestial Bodies*

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1. Introduction

For explanation of the axial rotation of the celestial bodies several hypotheses are proposed. Shmidt [1] considers the direct rotation of the planets and of the Sun as a result of the falling meteoric particles upon them. Artemyev and Radzievskii [2] assume that the meteoric particles falling on the planet transfer their positive momentums only and turn it directly about its own axis. Recently it was discovered, however, that Venus has a retrograde rotation. The planet Uranus is also rotating retrogradely. Alfven considers the axial rotation of the celestial bodies as a result of the action of magnetic forces. These hypotheses are not satisfactory and the phenomenon is still a topical problem. In this paper a new explanation of this phenomenon is proposed.

2. Method and Results

We proceed from the following original experiment. A homogeneous sphere with radius r is circulating in an circumference with radius r_1 lying in a horizontal plane. The sphere can rotate freely about its own axis perpendicular to the plane of the circumference. The system has two degrees of freedom. Let us designate by Ω_1 the initial angular velocity of the circular motion of the sphere, by Ω_2 the angular velocity of its circular motion at the end of a certain interval of time ΔT and by ω the angular velocity of the proper axial rotation of the sphere. We observe that

1. At $\Omega_1=\Omega_2=$ const, i. e. at a uniform circular motion $\omega=0$; the sphere does not rotate about its own axis.
2. At $\Omega_1>\Omega_2$, i. e. at a decelerate circular motion $\omega>0$; the sphere obtains a *direct* axial rotation.
3. At $\Omega_1<\Omega_2$, i. e. at an accelerate circular motion $\omega<0$; the sphere obtains a *retrograde* axial rotation.

*For open discussion.

It is not difficult to see that the axial rotation of the sphere is caused by the action of the unequal forces of inertia of the particles of the sphere which appear at the change of the angular velocity of its circular motion. The value of the forces of inertia of the particles of the sphere depends on their distance to the centre of the circumference and on the difference $|\Omega_1 - \Omega_2|$.

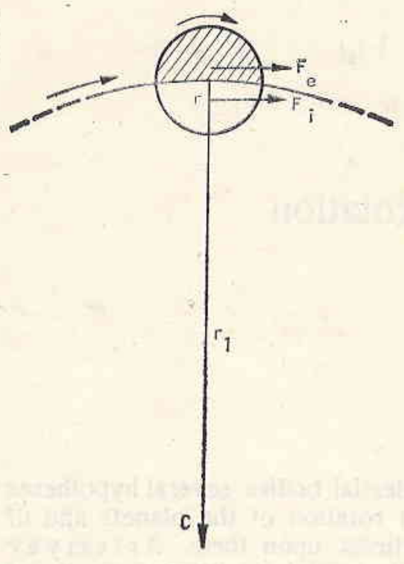

Fig. 1

The angular velocity of the circular motion of the sphere can be changed by direct action or by the change of the radius of the circumference from r_1 to r_2.

We have to determine the angular velocity of the axial rotation of the sphere ω obtained at the change of the angular velocity of its circular motion from Ω_1 to Ω_2.

We solve this problem in the following way. The cylindrical surface passing through the circumference and the rotational axis of the sphere divides the sphere in two parts: external one with mass m_1 and internal one with mass m_2, as shown in Fig. 1. We solve the problem at $r_1 \gg r$ so that we could assume $m_1 = m_2 = m$. A similar case we have, for example, at a spherical artifical Earth's satellite. The part of its orbit limited by the satellite is almost a straight line.

We designate by F_e and F_i the resultants of the particles' forces of inertia of the external (dashed) and internal hemispheres. We assume with sufficient accuracy that at $r_1 \gg r$ the fulcrums of the resultants F_e and F_i are at a distance equal to $3/8\,r$ from the centre of the sphere. We designate the force, which turns the sphere about its axis (directly at $\Omega_1 > \Omega_2$, as shown in Fig. 1, and retrogradely at $\Omega_1 < \Omega_2$) by F.

Obviously

(1) $$F = F_e - F_i,$$

or

(2) $$F = m a_e - (-m a_i),$$

where a_e and a_i are the tangential accelerations of the mass-centres of the external and internal hemispheres.

At the change of the angular velocity of the circular motion of the sphere from Ω_1 to Ω_2 the linear speed of the mass-centre of the external hemisphere changes from V_{e1} to V_{e2}, and that of the internal hemisphere from V_{i1} to V_{i2}. At the same time the mass-centre of the external hemisphere, where the fulcrum of the force F is formally assumed, passes a path $S = S_{e2} - S_{e1}$. Multiplying both parts of (2) by ds and integrating in the above limits we obtain

(3) $$\int_{S_{e1}}^{S_{e2}} F\,ds = \left| -m \int_{V_{e1}}^{V_{e2}} v\,dv - \left(-m \int_{V_{i1}}^{V_{i2}} dv\right) = m \int_{V_{e2}}^{V_{e1}} dv - m \int_{V_{i2}}^{V_{i1}} dv \right|,$$

(4) $$A = \left| \left(\frac{mV_{e1}^2}{2} - \frac{mV_{e2}^2}{2}\right) - \left(\frac{mV_{i1}^2}{2} - \frac{mV_{i2}^2}{2}\right) \right|,$$

where A is the work of the force F in the path S.

The kinetic energy E of the obtained axial rotation of the sphere is equal to the work of the force F, i. e. $E=A$

$$E=\frac{2}{5}X\frac{2mr^2\omega^2}{2}, \tag{5}$$

where $2m$ is the mass of the sphere. From (4) and (5) we obtain

$$\frac{4r^2\omega^2}{5}=|(V_{e1}^2-V_{e2}^2)-(V_{i1}^2-V_{i2}^2)|; \tag{6}$$

$$V_{e1}=\Omega_1(r_1+3/8r); \quad V_{e2}=\Omega_2(r_1+3/8r);$$

$$V_{i1}=\Omega_1(r_1-3/8r); \quad V_{i2}=\Omega_2(r_1-3/8r).$$

With the above values of V_{e1}, V_{e2}, V_{i1} and V_{i2} in (6) we obtain

$$\pm\omega=\sqrt{\frac{15r_1}{8r}|(\Omega_1^2-\Omega_2^2)|}. \tag{7}$$

If the change of the angular velocity of the circular motion of the sphere is performed by changing the radius of the circumference from r_1 to r_2, formula (7) takes the form

$$\pm\omega=\sqrt{\frac{15}{8r}|(\Omega_1^2 r_1-\Omega_2^2 r_2)|}. \tag{8}$$

The relations (7) and (8) are valid in an uniform field of gravitation. For a homogeneous spherical satellite moving in a non-uniform field of gravitation, however, (4) has the form

$$A=E=\left|\left[\left(\frac{mV_{e1}^2}{2}+E_p^{e1}\right)-\left(\frac{mV_{e2}^2}{2}+E_p^{e2}\right)\right]-\left[\left(\frac{mV_{i1}^2}{2}+E_p^{i1}\right)-\left(\frac{mV_{i2}^2}{2}+E_p^{i2}\right)\right]\right|, \tag{9}$$

where E_p^e and E_p^i are the potential energies of the external and internal hemispheres of the satellite. At our laboratory experiment we have $E_p^{e1}=E_p^{e2}=E_p^{i1}=E_p^{i2}=mgh$, so that in this case (9) is reduced to (4).

The total energy of a homogeneous spherical Earth's satellite moving in a circular orbit is

$$E_k+E_p=\frac{GM2m}{2r_1}+GM2m\left(\frac{1}{R}-\frac{1}{r_1}\right)=GM2m\left(\frac{1}{R}-\frac{1}{2r_1}\right),$$

where M is the mass of the Earth, R is the average radius of the Earth, r_1 is the distance of the satellite to the centre of the Earth. For such a satellite passing from one orbit to another one (9) takes the form

$$\frac{2mr^2\omega^2}{5}=GMm\left|\left\{\left[\left(\frac{1}{R}-\frac{1}{2(r_1+3/8r)}\right)-\left(\frac{1}{R}-\frac{1}{2(r_2+3/8r)}\right)\right]\right.\right. \tag{10}$$

$$\left.\left.-\left[\left(\frac{1}{R}-\frac{1}{2(r_1-3/8r)}\right)-\left(\frac{1}{R}-\frac{1}{2(r_2-3/8r)}\right)\right]\right\}\right|.$$

From (10) we obtain

$$\pm\omega=\sqrt{\frac{15}{16r}\left|\left(\frac{GM}{r_1^2-(3/8r)^2}-\frac{GM}{r_2^2-(3/8r)^2}\right)\right|}. \tag{11}$$

The total energy of a homogeneous spherical Earth's Satellite moving in an eliptical orbit is

$$E_k+E_p=\frac{GM2m\left(\frac{2}{r_1}-\frac{1}{a}\right)}{2}+GM2m\left(\frac{1}{R}-\frac{1}{r_1}\right)=GM2m\left(\frac{1}{R}-\frac{1}{2a}\right),$$

where a is the major semi-axis of the satellite's orbit. In this case we obtain from (9) in a similar way the following relation:

$$(12)\qquad \pm\omega=\sqrt{\frac{15}{16r}\left|\left(\frac{GM}{a_1^2-(3/8r)^2}-\frac{GM}{a_2^2-(3/8r)^2}\right)\right|}.$$

At the movement of the the satellite in one orbit only we should have $a_1=a_2=\text{const}$, $E_k+E_p=\text{const}$ and $\omega=0$, i.e. in such a case the axial rotation of the satellite does not depend on its orbital motion.

For a homogeneous cylindrical satellite with moment of inertia $I=2m\left(\frac{r^2}{3}+\frac{z^2}{4}\right)$, we obtain the following relation

$$(13)\qquad \pm\omega=\sqrt{\frac{r}{2\left(\frac{r^2}{3}+\frac{z^2}{4}\right)}\left|\left(\frac{GM}{a_1^2-(1/2r)^2}-\frac{GM}{a_2^2-(1/2r)^2}\right)\right|},$$

where r is the rotational radius of the satellite equal to $H/2$, z is the radius of its cross-section. Passing from one orbit to another the satellite turns as an aircraft propeller, in the plane of its orbit. (We neglect the precession.)

The formulae (11), (12) and (13) give the angular velocity of a non-stabilized satellite if in the first orbit $\omega=0$. If in the first (initial) orbit $\omega>0$ or $\omega<0$, formulae (12), and (11) and (13) as well, should be written as follows:

$$(12a)\qquad \pm\Delta\omega=\sqrt{\frac{15}{16r}\left|\left(\frac{GM}{a_1^2-(3/8r)^2}-\frac{GM}{a_2^2-(3/8r)^2}\right)\right|},$$

where $\Delta\omega$ is the increment (positive or negative) of the angular velocity of the axial rotation of the satellite during a certain interval of time ΔT, during which a_1 changes to a_2.

As $a^2\gg(3/8r)^2$, formulae (12), and (11) and (12a) as well can be written in this form

$$(14)\qquad \pm\omega=\sqrt{\frac{15}{16r}\left|\left(\frac{GM}{a_1^2}-\frac{GM}{a_2^2}\right)\right|}.$$

It is known that the light pressure provokes an essential perturbation of the orbit of the light spherical satellites at a height >700 km. By such a satellite we could verify formula (14). Determining the major semi-axis of its orbit in the beginning and in the end of a certain interval of time ΔT, we calculate ω by formula (14) and compare the result with the increment of ω during the same interval ΔT obtained by formula (15)

$$(15)\qquad \pm\Delta\omega=|\omega_1-\omega_2|,$$

where ω_1 and ω_2 are the observed angular velocities of the axial rotation of the satellite in the beginning and in the end of the same interval ΔT. The results obtained could be verified by a special satellite launched for this purpose if it is possible to change the major semi-axis of its orbit in desired values.

The relations (7) and (8) could be verified under laboratory conditions.

A synchronous change of the orbital period and the period of the axial rotation of the second Soviet satellite (1957β) and the last stage of the third Soviet satellite (1958δ_1) with the change of the solar activity has been really established [3]. The observed increase of the rotational period about the centre of mass of the last stage of the third Soviet satellite during its spiral approach to the Earth (from 8 to 9.5 s) [4] can be also explained by relation (13). The atmospheric resistance is believed to play a secondary role here. This factor is negligible for the massive satellites moving in the upper atmosphere.

At the verification of the relations obtained we must have in mind that the vector of the last push of the rocket does not always pass through the centre of mass of the satellite and would turn it about that centre [5]. Consequently, we must take the initial conditions into account.

The calculations by formula (12) show that a homogeneous spherical satellite, e. g. with radius $r=8$ m, passing from one orbit with major semi-axis $a_1=6{,}500$ km to a higher one with $a_2=6{,}650$ km, at initial condition $\omega=0$, turns directly about its centre of mass with angular velocity $+\omega=0.222$ rad/s, or with a period $t=28$ seconds. The same result but with retrograde rotation ($-\omega=0.222$ rad/s) should be obtained if this satellite passes from the higher to the lower orbit.

It is known that the equations of the passive flight of the artificial cosmic objects do not, as a matter of principle, differ from the equations of the motion of the natural celestial bodies. For this reason we could say that the relation (12) should be valid for the celestial bodies as well. The difference will be only in the corrections depending on the corresponding moment of inertia and the inclination of the rotational axis of the body to the plane of its orbit.

In rigid body dynamics it is assumed that the rotational motion of spherical and homogeneous bodies does not depend on their advance motion. The differential equations of the translatory motion and that of the rotational motion of the body about its centre of mass are considered separately. This statement is true, as we have seen, at rectilinear motion and at uniform circular motion of the body. It is not true at non-uniform circular or curvilinear motion of the body. The different forces of inertia of the particles of the body which appear in such a case have not been taken into account.

An important role for the difficulties arising at the solution of the problem of the axial rotation of the celestial bodies is played by the theorem of Lagrange-Laplace for the stability of the solar system. We have seen that at $a=$ const the axial rotation of the satellite does not really depend on its orbital motion. The investigations of the series which are used in celestial mechanics in the theory of perturbations show, however, that they are divergent as a rule. The theorem of Lagrange-Laplace is not strictly proved. A. Friedman has proved that the Universe is periodically expanding and contracting [7]. The expansion of the Universe was confirmed by observation (Hubble's law). According to R. Zaikov the Sun has orbital acceleration [8, 9]. The inner satellite of the planet Mars, Phobos, also has orbital acceleration [6]. Consequently, the solar system is not stable during long periods of time lasting millions and billions of years.

According to the hypothesis of V. G. Fessenkov the planets of the solar system had been formed far nearer to the Sun, in comparison with their present distance. Consequently, during the first period of their existence the planets had been moving off in spirals from the Sun, and according to relation (12) they obtained their direct axial rotation. The observed retrograde rotation of the planet Venus could be explained with the same relation (12) if this planet has orbital acceleration. According to N. Bonev the young planet should have a retrograde axial rotation thanks to the Keplerian distribution of the velocities of its particles [9]. Such an explanation of the retrograde rotation of

Venus does not contradict our theory. According to (12), at the moving off from the Sun the unequal forces of inertia of the perticles of the planet could brake the retrograde rotation of the planet and turn it directly in the course of time. During the second period of existence of the planet, when it approaches the Sun, the forces of inertia which are acting in the opposite direction could brake the direct rotation of the planet and turn it retrogradely. It is necessary to know how the major semi-axis of the orbit of Venus is changing. This will show us whether Venus is a young or old planet.

The transition of the direct axial rotation of the celestial body to a retrograde rotation is probably achieved simultaneously with the change of the inclination of its rotational axis to the normal to its orbit plane. At an inclination of $>90°$ the direct rotation turns to retrograde. The inclination of the planet Uranus, which is rotating retrogradly, is 98°. The inclination of Venus is $\approx 90°$ (probably $>90°$). The Moon is in a state of gravitational stabilisation.

Formula (12) or (14) make it possible to determine approximately at what distance from the Sun the planets had been formed. For our Earth, for example, at $M=2\times10^{33}$ g (the mass of the Sun), $a_2=1.496\times10^{13}$ cm, $\omega=7.292\times10^{-5}$ rad/s, formula (14) gives $a_1=56$ millions of kilometres. This result is in a very good agreement with Fessenkov's hypothesis.

The orginal experiment with the homogeneous sphere giving a qualitaive explanation of the axial rotation of the celestial bodies, which is the base for our investigation of this phenomenon, was performed by the author's father Rasho Tilchev in 1932.

Acknowledgement. I am very grateful to the staff of the Central Laboratory for Space Research of the Bulgarian Academy of Sciences for the kind support in publishing my paper.

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Решение проблемы о вращении небесных тел

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(Резюме)

В этой статье дается теоретическое объяснение экспериментально установленной зависимости между неравномерным круговым поступательным движением однородной сферы и ее собственным осевым вращением. На основании полученной зависимости предлагается оригинальное решение проблемы осевого вращения небесных тел и неориентированных спутников Земли.